Energy, Mines and Resources Canada | Énergie, Mines et Ressources Canada

CANMET

Canada Centre for Mineral and Energy Technology | Centre canadien de la technologie des minéraux et de l'énergie

ON THE MODELLING OF METAL FORMING PROCESSES - A REVIEW

J.J.M. TOO
METAL FORMING SECTION

MARCH 7, 1983

This document is an unedited interim report prepared primarily for discussion and internal reporting purposes. It does not represent a final exposition of the opinion of the Canada Centre for Mineral and Energy Technology (CANMET)

Ce document est un rapport provisoire non-révisé et rédigé principalement pour les besoins de discussion et de documentation interne. Il ne représente nulle façon l'opinion définitive du Centre canadien de la technologie des minéraux et de l'énergie (CANMET)

26 pp

MINERALS RESEARCH PROGRAM
PHYSICAL METALLURGY RESEARCH LABORATORIES
REPORT MRP/PMRL 83-19(TR)

MICROMEDIA
AUG 19 1983

PHYSICAL METALLURGY RESEARCH LABORATORIES

REPORT MRP/PMRL 83-19(TR)

ON THE MODELLING OF METAL FORMING PROCESSES - A REVIEW

by

J.J.M. TOO

ABSTRACT

This report reviews the current state of the art of the finite element method and its related subjects in the modelling of metal forming processes, and identifies research areas which are necessary for further understanding of the behaviour of metal throughout the forming process.

RÉSUMÉ

Ce rapport examine l'état actuel de nos connaissances sur la méthode des éléments finis et les sujets reliés à la modellisation des processus de mise en forme des matériaux, et identifie les domaines où la recherche est nécessaire afin d'améliorer notre compréhension du comportement des métaux au cours des processus de formage.

INTRODUCTION

Metal forming technology has been in existence for many centuries. Significant progress made over the years on metal forming technology has relied mainly upon cumulated experience of trial-and-error methods. These methods often result in waste of materials, energy and time. However, because of the complex nature both of metals and in the processes involved, empirical methods will still remain as an important approach to metal forming for a long time.

Analytical modelling of metal forming processes started to gain recognition in the 1930's (1-4). It has now been developed extensively because of the call for energy conservation, precision, and for minimizing the cost of production.

Recently, the Finite Element Method has gained widespread recognition in the modelling of metal forming. This is because the method is flexible and versatile in handling boundary conditions, complex material properties and geometry. While the use of the method has been successful in some metal forming problems (for example, Ref. 5 and 6), difficulties relating to the nature of high nonlinearity remain to be resolved. The source of these difficulties can be traced to the lack of an appropriate mathematical description of a constitutive relation; and the variation of the boundary conditions and the intrinsic characteristics of metal with time, in many processes. The physics that dominates these changes is not yet fully understood, and mathematical descriptions of the fundamental physics still remain in a state-of-the art.

It is important to point out that the Finite Element Method is merely a tool for solving mathematical equations by which certain physical behaviour can be described. The success of using the Finite Element Method in the field of behaviour modelling is thus very much dependent upon the correctness or the closeness of the mathematical description of the physics relating to the problem being modelled.

The objective of this report is to review the current state-of-the-art of the Finite Element Method as well as its related subjects in the modelling of metal forming processes; thereby to establish and identify research areas that are necessary for further understanding of the behaviour of metals throughout the forming process. Hopefully, these will lead to a reduced dependency on empiricism, and to a model for the use of product control in metal forming.

Several review papers on the modelling of metal forming processes have been written by various authors, and very thorough discussions have been made with respect to types of metal forming processes (7,8). Since most of metal forming processes share similar deformation mechanisms and yet encompass a wide range of disciplines of knowledge, the discussions in this report will be directed toward the evaluation of the capability of modelling physical phenomena in the light of the current Finite Element Method and related technology, rather than types of forming processes.

Among the wide range of disciplines of knowledge, only the salient physical aspects, such as interfacial friction and heat conduction, thermal/mechanical coupling, elasto-plastic behaviour, phase transformation, and microstructures will be discussed in order.

FRICTION AND HEAT CONDUCTION AT CONTACTING SURFACES

Friction is one of the most important factors in almost all metal forming processes that can seriously affect the forming performance. For example, it can cause die wearing, scoring of finished products, die/workpiece sticking, and slowing down or prevention of the deformation process. In a forging process, the distribution of pressure can be increased because of the increase of friction. Lubricants have been employed for obtaining greater metal formability and for better control on surface quality of final products. The hydrodynamics of lubricants, and the relation between the pressure distribution and the surface friction as well as the metal structures of final

products must be fully understood in order to minimize the waste of materials, and to optimize the cost of production.

Frictional behaviour and contact problems have been modelled by various investigators (9-12) on a continuum basis using the Finite Element Method. In these models, it is assumed that full contact is achieved when two surfaces meet. In reality, because most surfaces are microscopically rough, only a discrete number of asperities meet in a random manner. The real contact area is thus a fraction of the nominal contact area. Models based on nominal contact area can thus be seriously in error.

The importance of surface roughness effects on contacting surfaces has been recognized only in the last two decades. Since then studies on surface characteristics have been made using a statistical approach. Various types of distribution functions are proposed (14-16), but most of the work that has been done is one-dimensional.

The hydrodynamics of lubricants has been studied by Reddi (17), Argyris and Scharp (18), Taylor and O'Callaghan (19) and Oh and Huebner (13), using Finite Element Methods. The use of Finite Element Methods has circumvented some of the boundary condition problems that are often encountered in the finite difference method. However, the effect of surface roughness is not effectively accounted for. More recently, Elrod (20) presented a formulation for thin film lubrication between a smooth surface and a rough surface in relative motion. A mean roughness and a global Reynold's equation have been derived but they have been limited to the application of striated surfaces.

Recently, Too and Wu (21) extended the statistical approach proposed by Pullan and Williamson (15) and Christensen (22) and Christensen and Tonder (23) for the modelling of lubricated interfaces using the Finite Element Method. A truncated probability function is used instead of the approximate polynomial functions as used by others (14,15,22,23). In this formulation, it is assumed that all contact points are deformed plastically and the volume under the contacting interface is conserved. Elastic deformation at the contact is not considered. The validity of this proposed approach has yet to be confirmed experimentally.

The discrete nature of contact points in a nominally contacted area implies that similar constraints must be made with regards to heat conduction at an interface as those made with friction. In practice the nominal contact area is used for the calculation of contact pressure in the heat

transfer calculation. In reality, the real contact area is only a small fraction of the nominal contact area. The heat transfer coefficient is usually expressed as a function of the contact pressure. Thus, the use of nominal contact area in the calculations can cause serious errors both in the prediction of contact pressure and in the evaluation of heat transfer coefficients. Despite their inadequacy, models are formulated on the basis of nominal contact pressures and contact areas (9,11,24). This approach, in fact, allows qualitative predictions. However, quantitative prediction remains as an art of engineering intuition. Clearly, the application of this type of method to the modelling of metal forming should be avoided, especially to hot forming where temperature distribution during the forming process has a serious implication on the material properties of the final products.

More recently, statistical approaches have been introduced by Gupta and Cook (25) and Mikic (26), and Burde and Yovanovich (27). In their work, either elastic or plastic deformation of asperities is considered. The statistical approach and the inclusion of deformation effects represent a significant improvement on the physical description of the heat conducting interfaces. However, it should be realized that, because of the varied asperity heights and surface waviness, the pressure distribution is not uniform. Some asperities may deform elastically, others plastically. A combined elastic/plastic model must be formulated in order to model the heat conduction more precisely. However, this remains as a subject for future development.

THERMAL/MECHANICAL COUPLING

In hot-forming processes, heat conduction and mechanical deformation are coupled. As the heat dissipates from the workpiece, the geometries and the relative position of the die and workpiece can change because of thermal stresses; conversely as the die and the workpiece deform under applied loads, the temperature distribution also changes because the characteristics of heat conduction are strong functions of the boundary geometry. The change of temperature distribution can alter the mechanical properties of the workpiece, and consequently affect the mechanical deformation. Furthermore, a large plastic deformation is often accompanied by substantial heat dissipation as a result of internal friction. This can significantly affect the temperature distribution in, and the behaviour of, a workpiece. This coupled behaviour must be considered correctly when formulating a model.

Various analytical investigations (e.g., 28,29) have been conducted in the past by first calculating the deformation at a constant temperature and then evaluating the temperature distribution using the calculated deformed geometry; or conversely, by analyzing the heat conduction based on a fixed geometry throughout the whole heat transfer process, and then calculating the mechanical deformation using the temperature distribution results as a sequence of thermal loads and mechanical loads together. It is quite obvious that this type of method does not adequately represent the effects of thermal/mechanical coupling.

As an alternative, the coupled thermal/mechanical behaviour may be modelled using psuedo-coupling (30,31). Analyses were performed alternately between heat conduction and mechanical deformation in a step-by-step manner. The time domain is divided into discrete increments and the geometry of the solid is updated at each time increment. While this type of model is more descriptive than its predecessors, a drawback still exists because of the time lag between the thermal step and mechanical step. For situations where steep changes in thermal physical and mechanical properties prevail, large errors can occur.

A truly coupled thermal/mechanical analysis was presented by Argyris et al. and Oden et al. (32,33) that accounts for deformation and heat transfer simultaneously. In Oden's work, the material is assumed to be elastoplastic. His method was applied with some success to a three-dimensional bar constrained in one direction and heated at one corner. Zienkiewicz et al. presented a coupled formulation in problems of steady state extrusion and deep drawing (5). This formulation takes the plastic heat dissipation into consideration. It is based on the combination of a momentum equation, an equilibrium equation and an energy equation. The combination of the three equations results in a set of simultaneous equations consisting of flow and heat condition characteristics. It has been demonstrated that the formulation is quite efficient and adequate for the modelling of extrusion and deep drawing problems. However, the problem of obtaining a convergent solution remains to be resolved. Recently, Rebelo and Kobayashi (6) developed a coupled model based on a rigid viscoplasticity theory and the concept of internal parameters. The plastic strain and dislocation density are the only two internal parameters adopted. Comparisons of predicted results with experimental results on simple ring compression tests showed good agreement. However, further improvement

should be made in the identification of internal parameters that dominate the deformation mechanism in the forming process.

ELASTOPLASTIC BEHAVIOUR

In a uniaxial test, metal often exhibits linear stress/strain behaviour at low loads. As the load increases the stress/strain curve gradually departs from linearity. The material is said to yield at a point where the departure from linearity begins. The classical theory of plasticity is built on the generalization of this experimental observation to a three-dimensional stress state. The deformation pertaining to recoverable elastic behaviour is separated from that pertaining to irreversible plastic behaviour by a surface called the yield surface. Mathematically, this is expressed as a function of stresses:

$$f(\sigma_{ij}) = k^2 \ , \qquad \text{Eq 1}$$

where k is the yield stress, and σ_{ij} is the stress tensor. The elastic and plastic strains are often expressed in incremental forms. The total incremental strain is assumed to be the sum of the incremental elastic strains and the incremental plastic strains:

$$d\epsilon_{ij} = d\epsilon^{e}_{ij} + d\epsilon^{p}_{ij} \qquad \text{Eq 2}$$

This theory has been widely accepted and has become the tool for modelling various problems on plastic deformation. However, a number of difficulties often arise when applying this theory:

- The definition of yield stress is purely an idealization of material behaviour. In reality, yield stress is difficult to measure precisely. In fact, microscopically plastic deformation begins in a workpiece almost immediately upon loading. The analytical division of elastic and plastic components is merely a convenient artifice.
- As defined in Eq 1, the yield surface can be determined only if the yield stress is given. This makes the theory rather awkward to apply since the yield surface in reality is not only a function of stress, but also is a function of stress history. Though a stress hardening parameter can be in-

troduced to account for this history dependency, the question of how to determine this hardening parameter remains unsolved.

- The assumption made in Eq 2 is valid only for the case of a small deformation. It is not applicable for large deformation where elastic and plastic strains are coupled (i.e., the additive rule is no longer applicable).

Despite these difficulties, the classical plasticity theory has been applied both in the forms of rigid plasticity and elasto-plasticity in the modelling of metal forming processes (34-36). The reason for using rigid plasticity is that the elastic deformation is small in comparison with the large plastic deformation. Indeed, this approach has been very useful for the estimation of load or pressure for metal forming processes and it offered a practical way of qualitatively predicting metal behaviour. However, it is unable to give a correct description of the history-dependent behaviour as normally observed in the large plastic deformation. The classical theory of elasto-plasticity has been employed for the improvement on the description of history dependency (37-39). However it is valid only when the deformation is small. The application of this theory to the analysis of large deformation frequently results in oscillating shear stresses for a monotonically increasing shear strain. There is a coupling between elastic strain and plastic strain. This coupling is a result of the plastic flow that always occurs in a material already stressed to the yield point. A subsequent elastic strain increment can be unduced by incremental loading. These two components interact as the material deforms.

To eliminate the anomaly in the classical plasticity theory for small deformation, a nonlinear kinematic finite deformation theory has been proposed by E.H. Lee and his co-workers (40-44). In this theory the coupled interaction is expressed in terms of the deformation gradient $\underset{\sim}{F}$:

$$\underset{\sim}{F} = \underset{\sim}{F}^{e}\underset{\sim}{F}^{p} \qquad \text{Eq 3}$$

where $\underset{\sim}{F}^{e}$ is the elastic deformation gradient, and $\underset{\sim}{F}^{p}$ is the plastic deformation gradient in an unloaded state. The total strain is expressed as

$$\underset{\sim}{E} = (\underset{\sim}{F}^{T}\underset{\sim}{F} - \underset{\sim}{I})/2 \qquad \text{Eq 4}$$

Lee also proposed a modified definition of back stress for the description of kinematic hardening (44). It was pointed out that significant discrepancies exist between the results obtained using conventional strain and the new strain and kinematic hardening descriptions. For strains near 2, the stress results can differ by 40%, and the difference grows rapidly with increasing strains. This suggests the conventional approach should be carefully reassessed.

DESCRIPTION OF LARGE DEFORMATION

In the metal forming processes, because of the large deformation and rotation, as previously discussed, the ordinary description of strains based on small strain theory is no longer valid. For large deformation, several types of strain description were formulated. These strains are named Green strains, Almansi strains, and centred strains (45,46). The difference in these strain descriptions lies in the choice of reference frames.

Green strain refers to the material coordinates (or Lagrange coordinates) a_i; the Almansi strain refers to the spatial (or Eulerian) coordinates, X; and the centred strain refers to the intermediate coordinates, X_i [$= 1/2(a_i + X_i)$]. The mathematical expression for the three strains can be written as:

$$E_{ij} = 1/2\left(\frac{\partial u_i}{\partial a_j} + \frac{\partial u_j}{\partial a_i} + \frac{\partial u_k \partial u_k}{\partial a_i \partial a_j}\right) \qquad \text{Eq 5}$$

$$\Sigma_{ij} = 1/2\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{\partial u_k \partial u_k}{\partial x_i \partial x_j}\right) \qquad \text{Eq 6}$$

$$\epsilon_{ij} = 1/2\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \text{Eq 7}$$

where E_{ij}, Σ_{ij} and ϵ_{ij} are respectively the Green strain, Almansi strain and centred strain tensors. u_i is the deformation (or displacement) vector, and the subscript i varies from 1 to 3.

It should be pointed out that the linear appearance of Eq 7 is rather deceptive. Since x_i' are functions of u_i, Eq 7 is in fact nonlinear.

The significant characteristic of the three strain tensors is that they represent rotational effect and rigid body motion correctly. However, because of the nonlinear nature of Eq 5, 6 and 7, the rule of additive decomposition of strains is not applicable. For example, if u_i is considered as the sum of u_i^1 and u_i^2, then, in general

$$E_{ij}(\underset{\sim}{u}) \neq E_{ij}(\underset{\sim}{u}^1) + E_{ij}(\underset{\sim}{u}^2) \qquad \text{Eq 8}$$

Writing the incremental strain as,

$$\Delta E_{ij} = E_{ij}(\underset{\sim}{u}) - E_{ij}(\underset{\sim}{u}^1) \qquad \text{Eq 9}$$

then, in general, ΔE_{ij} do not vanish even if the body movement corresponding to u_i^2 is a rigid body motion. This, in fact, is consistent with the discussion made regarding the summation rule given by Eq 2.

Green strain is most commonly used of the three stain tensors because the independent variables a_i are given quantities and are thus more convenient to compute (47).

PHASE TRANSFORMATION

In certain forming processes involving heating and cooling, the workpiece often experiences a wide range of temperature changes with a different rate at various stages of the forming process. Phase transformation is likely to occur at certain temperatures. Since the temperature distribution is non-uniform and the rate of heat dissipation across a workpiece may vary, different phase constituents may form at different parts of the workpiece. Consequently this may cause non-uniform thermal/mechanical property changes. Indeed, surface layer and non-uniform structures resulting from die chilling and rapid cooling have been observed experimentally. It was also shown experimentally that phase transformation can be induced by dynamic loads or shock waves (49), and the rate of phase change can be increased by the presence of stresses (50). These interdependent phenomena in a phase transformation must be understood in order to gain full control of the material properties of the products.

The main mechanism that dominates most of the phase transformation phenomena is the thermodynamic diffusion process. Mathematically, it can be treated and described by the Gibb's free energy and the diffusion equation of mass flow in solids. The flow can be related to the random motion of atoms migrating through lattices. However, a complete solution of the diffusion equation requires a detailed knowledge of transport properties, stress gradients, temperature and temperature gradients, etc. Such information is usually not available or incomplete. Theoretical and experimental work needs to be done to provide more accurate predictions and data.

From the theoretical aspect, analytical models can be developed based on the rigorous treatment of thermodynamics and diffusion kinetics. The fundamental theory of phase transformation can be found in the work presented by Lazarus (51) and Roberts (52). More comprehensive coverage of the subject has been discussed in books authored by Porter and Easterling (53) and by Stanley (54).

A review work has been conducted by Hillert (55) on methods of coupling phase equilibrium data with thermodynamic treatment. This type of method always chooses a mathematical expression for the representation of Gibb's free energy, expressed in terms of physical parameters for each individual phase. The available data are used to calculate these parameters in the free energy expression of each phase. The total free energy of the system is obtained by summing up the free energy of all phases. The equilibrium composition is then obtained by the minimization of the total free energy of the system. This type of method is particularly useful for determining the interfaces of multicomponent phases in a phase diagram, where experimental data are insufficient at the interfaces. The method appears to be similar to the conventional curve fitting technique. However, the parameters used do possess a physical interpretation. A similar approach has been applied by Uhrenius (56) for the analysis of equilibria between austenite, ferrite and carbides. It was found that, in general, the analyses were more accurate at high temperatures than at lower temperatures. Kirkaldy et al. (48) studied the effect of dilute alloy addition on the Fe-Fe_3-C phase diagram using a two phase formula, and developed a computer program for the calculation of TTT and CCT diagrams. Comparison of results with available data for commercial steels has shown good agreement.

In determining phase changes, the distribution of temperature, stresses, and strains in the workpiece must be known at all stages of the process. A thermal/mechanical, finite element model can be used for obtaining this information. In fact, the modelling of phase transformation using the Finite Element Method has been considered by Too and Trafford (57). The work requires that a phase diagram and a set of thermal/mechanical property data must be known. For the case where a phase diagram is incomplete, interpolated values must be used.

The method presented by Inoue and Wang (58) for the modelling of coupled behaviour of stress, temperature and phase changes is noteworthy of mention. In this method, a stress and temperature dependent function is introduced for the evaluation of the volume fraction of phase constituents. This volume fraction is then related to the plastic potential. The coupled thermal/mechanical behaviour and phase transformation problem is solved using the Finite Element Method. This method has formed a truly-coupled framework for dealing with thermal/mechanical and phase-transformation problems. It is the reviewer's opinion that this method has potential for resolving the coupled phase transformation phenomena. It can be developed further by introducing rigorous, thermodynamic theory into the stress and temperature dependent function for a more precise physical description.

CONTINUUM MECHANICS AND MICROSTRUCTURES

As a piece of material is subjected to a set of loads and/or temperatures, it undergoes a sequence of changes both in geometrical configuration and in material properties. The study of this type of change is within the realm of continuum mechanics. Continuum mechanics treats a material as a continuous media. The analysis is centred on the macro(or apparent) structural deformation. It usually assumes that material properties can be obtained from data gathered from simple experiments. The constitutive relation required for the analysis is normally constructed on these data bases. The continuum mechanics approach works well when material properties are relatively independent of the change in geometrical configuration or when the material properties can be characterized for all conditions. Difficulties start to emerge when properties and deformation are mutually dependent (or coupled). For example, in the case of a large elasto-plastic deformation, it is rather difficult to derive a meaningful constitutive equation purely based upon simple ex-

perimental data, yet more rigorous experiments are rather difficult to perform. The lack of an appropriate constitutive relation has made the continuum mechanics approach inaccurate. This urged the necessity of a more rigorous approach for deriving constitutive relations. Clearly, this cannot be achieved without a good understanding of the fundamental mechanisms of the microstructural effects.

It has been recognized for some time that several important mechanisms are responsible for the deformation of polycrystal materials. At low temperatures, polycrystals deform by dislocation glide (59). The stress required for the glide is set by the lattice resistance. It is this resistance that accounts for the yield strength of most polycrystals. However, for soft metals, the yield strength is determined by the slip-over obstacles such as impurities, solutes, dislocations and grain boundaries. At high temperatures, when stress is high, the deformation mechanism usually involves grain boundary sliding and dislocation motion within the grains (60). At a lower stress, the deformation mechanism is mainly the diffusional flow resulting from the motion of particles from compressive grain boundaries towards the boundaries under tension (61). At very high temperatures, the deformation can be accompanied by recrystalization and formation of new grain structures.

Theoretically, these deformation mechanisms can be described mathematically on the basis of a micro scale using thermodynamics and statistical mechanics. This, in fact, has been studied by material science workers (62, 63). However, the computation time required for this type of approach is astronomical and prohibitive for any practical application. A more plausible approach is to bridge the gap between continuum mechanics and microstructures. The link between the two can be constructed through the constitutive relation modelling.

This approach has been pioneered by Rice, and Hill and Rice (64-67), and then Rice and Needleman (69) and Neal (70). The initial work was based on the second invariant flow theory with smooth yield surface and deformation theory. As commented by Pecherski (71), they do provide some satisfactory qualitative predictions of plastic instability. However, quantitative results (e.g., the critical strain pertinent to the onset of localization or the inclination of angle of shear bands to the maximum principal stress) do not agree with the experimental observation. The effect of local lattice rotations within shear bands was considered by Asaro (72) and Chang and Asaro

(73). They assumed the lattice rotation contains both rotation of material fibres and lattice vectors caused by elastic straining and rigid body rotation.

In a later work (68) Rice related the macroscopic plastic strain increment with a set of discrete internal variables and thermodynamic forces on a dislocation line. Each discrete variable and the thermodynamic force forms a conjugate pair in the formulation of a potential function. These internal variables are expressed in terms of local stresses and Burger's vectors. The rate of structural reorientation is assumed to depend on the macroscopic stress only through the conjugate thermodynamic force. A macroscopic flow potential is derived on this basis. The macroscopic potential is interpreted as the volume average of local flow potentials for a slip system of each individual crystallite of the material.

Independent investigations have also been performed by Valnis (74) and Valnis and Lee (75) on a similar basis. In these works, it is assumed that the free energy is a function of deformation gradient, temperature and internal variables. The last work of Valnis and Lee also assumes that the internal structure of the material is so disarranged that it presents no residual memory of its reference configuration. The material thus behaves as a fluid. However, the state of stress is determined by the history of deformation. An intrinsic time scale is proposed that accounts for the dependency of stress on history. It has been demonstrated by Valnis and Lee that this theory works well for the cases studied in this paper. It should be mentioned that in this fomulation, stresses are defined as the derivative of free energy with respect to strains. This is different from the classical definition.

Pecherski (71) proposed a model that accounts for lattice misorientation and rigid body rotation for a simple crystal. In this method, the deformation gradient is decomposed into the elastic and viscoplastic parts similar to the work by E.H. Lee, as discussed in section 4. A total Burger's vector increment is related to the continuum measures of viscoplastic deformation and lattice misorientation. The rate of change of the Burger's vector increment produced by plastic glide is then expressed in terms of geometrical configuraion. This concept can be extended for application to polycrystalline materials.

An interesting work by Tirosch and McQuintock (76) deserves special attention. It considers that plastic strains can be expressed by the super-

position of dislocation dipoles. The method is closely tied with the actual physics of the dislocation mechanism of plastic flow. According to the authors, applications to elastic-plastic and rigid plastic problems have yielded encouraging results.

More recently, Abe and Nagaki (77) studied the plastic deformation of polycrystals and slips in crystal grains using the Finite Element Method. The study is of preliminary nature and has encountered difficulties in obtaining convergent results. However, the approach has opened a new area for the study of polycrystal deformation.

CONCLUSIONS

The state-of-the-art in the modelling of metal forming has been reviewed above. The discussion has been centred on the modelling of the significant aspects that are common to all forming processes. Capabilities of, and the difficulties related with, each approach in the literature have been addressed. Hopefully they will provide us with a useful base in the planning for a future model development program.

Despite the difficulties encountered, analytical models have been very useful in helping to gain an insight into the physical properties of metals, and have helped to uncover problems that may not otherwise have come to light. The adoption of the Finite Element Method has opened a new horizon for the metal forming industry. Problems of a complex nature, that were considered unapproachable can now be attempted by using this method. The work of Rice and others (65-77) has offered possible ways of establishing a link between continuum mechanics and microstructures. However, the available technology is still far from being able to precisely predict metal behaviour. Further development is needed if models are to be used as a tool for the control of microstructures and properties of the material, and the control of the precise geometrical configuration of the final products. From the above discussion, it has become apparent that the problem of modelling metal forming represents a challenge to all researchers, not only because it encompasses a wide range of disciplines, but also the complexity involved with each. It can also be seen that almost all the concerned subjects are closely related with, and can be tackled by, the fundamental theory of thermodynamics, whether it is on the constitutive relation, or on the phase transformation, or on the

thermo-mechanical deformation and surface contacts. Clearly, analytical tools are available, at least for simple systems such as single crystal, simple phase transformation, etc. Although this type of simple system is nonexistent in the real world of metals, the methods can be extended to handle more complicated systems if important parameters can be identified through the physical understanding.

Regarding the use of Finite Element Method, two attractive features of the method should be noted:

1. The method is formulated on the basis of variational principles. The energy of a system (or a functional) is minimized for obtaining an equilibrium or a quasi-equilibrium state. Conceptually, this is consistent with the fundamental principle of thermodynamics. Therefore, it is most suitable for the rigorous treatment of the behaviour modelling of metals.

2. The method treats a media as an assembly of discrete elements. This may prove to be an advantage when dealing with microstructures, such as grain orientation, phase interfaces, etc. where they themselves are in fact discrete elements.

Regarding the modelling of constitutive relations, we know that metals are microscopically anisotropic. Impurities, vacancies, dislocations and grain sizes are distributed in a random manner. Any attempt of using a deterministic approach will likely be unfruitful. One area which is less well known in the field of modelling of metal forming is statistical mechanics. By its very nature, it should be considered as one of the most suitable approaches for the formulation of the constitutive relation. The same argument can be applied to the modelling of phase transformation which is in fact well established theoretically. However, further development is needed for more practical and realistic applications.

To summarize, the author proposes that:

1. A statistical approach should be applied in the modelling of friction and heat conduction at contacting surfaces, in constitutive relations, and in phase transformation.

2. A truly coupled thermo-mechanical model based on a large strain theory must be adopted.

3. Preferably the updated Lagrange approach should be used for efficient computation. The choice of strain definition should be carefully studied.

4. The Finite Element Method should be used as a mathematical tool in constructing the framework of the model because of its special features, as previously discussed.

5. The fundamental physics of the important parameters must be modelled as correctly as possible. Incorrect physical description can often cause oscillating and non-converging results.

Finally, the author wishes to emphasize that experimental studies must be performed in parallel with the theoretical modelling, if the model is to be capable of quantitative prediction.

REFERENCES

1. Sachs, G.; Zeits Ang Math Mech 7; 1927.

2. Hitchcock, J.H.; Am Soc Mech Eng Research Publication; 1930.

3. Hill, R., Lee, E.H. and Tupper, S.J.; Proc 6th Int Cong App Mech; Paris; 1946.

4. Sachs, G. and Klinger L.J.; Jour App Mech 14, A-88; 1947.

5. Zienkiewicz, O.C., Onate, E. and Heinrich, J.C. "Plastic flow in metal forming"; ASME AMD, vol 28; 1978.

6. Rebelo, N. and Kobayashi, S. "A couple analysis of viscoplastic deformation and heat transfer - I. Theoretical consideration, II. Applications"; Int J Mech Sci, vol 22; 1980.

7. Johnson, W. and Sowerby, R. "Metal forming processes: analysis and technology"; Applications of Numerical Methods to Forming Processes; ed. H. Armen and R.F. Jones, Jr.; AMD, vol 28; Dec. 10-15, 1978.

8. Shabaik, A.H. "Mechanics of plastic deformation in metal forming processes experimental and numerical methods"; Current Advances in Mechanical Design and Production; Proc of the 1st Int Conf, Cairo Univ., Egypt; Dec. 27-29, 1979. Edited by G.S.A. Shawki and S.M. Metwalli; Pergamon Press; 1980.

9. Too, J.J.M., Hsu, T.R. and Bertels, A.W.M. "FULMOD An inelastic analysis program to predict the operating behaviour of CANDU fuel elements" Fuel Element Analysis, ASME Special Publication; ed. Y.R. Rashid and F.C. Weiler; 1975.

10. Hughes, T.J.R., Taylor, R.L., Sackman, J.L., Curnier, A. and Kanoknukulchai, W. "A finite element method for a class of contact-impact problems"; Computer Methods in Applied Mechanics and Engineering, vol 8; 1976.

11. Rigby, G.L., Bean, J.D. and Too, J.J.M. "A model for thermal-mechanical behaviour across a small gap"; presented at AIAA/ASME Fluid, Plasma, Thermophysics and Heat Transfer Conference, June 8-11, 1982; St. Louis, MO; also to be issued as AECL-7470; 1982.

12. Jain, S.C., Sinhasan, R. and Singh, D.V. "EHD-analysis of plain slider bearing having flexible porous bearing pad"; Numerical Methods for Coupled Problems; ed. E. Hinton, P. Bettess and R.W. Lewis; Pineridge Press; Swansea, U.K.; 1981.

13. Oh, K.P. and Huebner, K.H. "Solution of the elastohydrodynamic finite journal bearing problem"; Journal of Lubrication Technology; Transaction of the ASME; July, 1973.

14. Nayak, P.R. "Random process model of rough surfaces"; Transaction of the ASME, p 398-407; July 1971.

15. Pullen, J. and Williamson, J.B.P. "On the plastic contact of rough surfaces"; Proc of Royal Soc of London, A327, p 159-173; 1972.

16. Gupta, P.K. and Cook, N.H. "Statistical analysis of mechanical interaction of rough surfaces"; Journal of Lubrication Technology, p 19-26; Jan. 1972.

17. Reddi, M.M. "Finite element solution of the incompressible lubrication problem"; J Lubric Technol; Trans Am Soc Mech Eng, vol 91 (Series F); 1969.

18. Argyris, J.H. and Scharpe, D.W. "The incompressible lubrication problem"; The Aeronautical Journal of the Royal Aeronautical Society, vol 73; 1969.

19. Taylor, C. and O'Callaghan, J.F. "A numerical solution of the elastohydrodynamic lubrication problem using finite elements"; J of Mech Eng Sci, vol 14 No. 4; 1972.

20. Elrod, H.G. "Thin-film lubrication theory for Newtonian fluids with surfaces possessing striated roughness or grooving"; Transaction of the ASME; J Lubric Technol, p 484-489; Oct. 1973.

21. Too, J.J.M. and Wu, C. "A stochastic-finite-element formulation for thin lubrication and gap heat conduction"; presented at Int Conf on Finite Element Methods; Shanghai, China; Aug. 2-6, 1982.

22. Christensen, H. "Stochastic models for hydrodynamic lubrication of rough surfaces"; Proc Inst Mech Eng, vol 184, p 1013-1022; 1969-1970.

23. Christensen, H. and Tonder, K. "The hydrodynamic lubrication of rough journal bearings"; Transactions of the ASME, p 166-172; 1973.

24. Boer, C.R. and Schroder, G. "Temperature in the die-billet zone in forging"; Annals of CIRP, vol. 30/1; 1981.

25. Gupta, P.K. and Cook, N.G. "Statistical analysis of mechanical interaction of rough surfaces" J Lubric Technol; Transaction of the ASME; Jan. 1972.

26. Mikic, B.B. "Thermal contact conductance; Theoretical considerations"; Int Journal of Heat and Mass Transfer, vol. 17, pp 205-214; Pergamon Press; 1974.

27. Burde, S.S. and Yovanovich, M.M. "Thermal resistance at smooth-sphere/rough-flat contacts: Theoretical analysis"; Thermophysics and Thermal Control; ed. R. Viskanta; vol. 65 of Progress in Astronautics and Aeronautics; 1978.

28. Johnson, W. and Kudo, H. "The use of upper-bound solution for the determination of temperature distributions in fast hot rolling and axisymetric extrusion processes"; J of Mechanical Sciences, vol. 1; 1960.

29. Tay, A.O., Stevenson, M.G. and Davis, G.V. "Using the finite-element method to determine temperature distributions in orthogonal machining"; Proc of the Institution of Mechanical Engineers; vol 188; 1974.

30. Banerjee, S., Bridges, A.J., Too, J.J.M. and Hsu, T.R. "A model for the analysis of fuel behaviour in transients"; Nuclear Engineering and Design; vol 42; 1977.

31. Nagpal, V., Lahoti, G.D. and Altan, T. "A numerical method for simultaneous prediction of metal flow and temperatures in upset forging of rings"; ASME paper No. 77-WA/PROD-35.

32. Oden, J.T., Bhandari, D.R., Yagawa, O. and Chang, T.J. "A new approach to the finite element formulation and solution of a class of problems in coupled thermoelasto-viscoplasticity of crystalline solids"; Nuclear Engineering Design; vol 24; 1973.

33. Argyris, J.H. and St. Doltsinis, J. "On the natural formulation and analysis of large deformation coupled thermomechanical problems"; Computer Methods in Applied Mechanics and Engineering; vol 25, No. 2; 1981.

34. Lee, C.H. and Kobayashi, S. "New solution to rigid-plastic deformation problems using a matrix method"; Transaction of ASME; Journal of Engineering for Industry; vol 95; 1973.

35. Chen, C.C. and Kobayashi, S. "Rigid-plastic finite-element analysis of ring compression"; Applications of Numerical Method of Forming Processes; ASME, AMD; vol 28; 1978.

36. Wang, N.M. "A rigid-plastic rate-sensitive finite element method for modelling sheet metal forming processes"; Numerical Methods in Industrial Forming Processes; ed. J.F.T. Pittmann, R.D. Wood, J.M. Alexander and O.C. Zienkiewicz; Pineridge Press, U.K.; 1982.

37. Wood, R.D. "The finite element method and sheet-metal forming"; Sheet Metal Industries; Aug. 1981.

38. Rao, S.S. and Kumar, A. "Finite element analysis of cold strip rolling"; Int J Mach Tool Des Res; vol 17; 1977.

39. Wifi, A.S. and Yamada, Y. "Finite element analysis of metal working processes"; 1st Cairo Univ. Conf in Mech Design and Prod; Advances in Mech Design and Prod; ed. G. Shawki and S. Metwalli; Pergamon Press; 1980.

40. Lee, E.H., Mallet, R.L. and McMeeking, R.M. "Stress and deformation analysis of metal forming processes"; Metal Forming Plasticity, Sym. Tutzing, Germany; Aug. 28-Sept. 3, 1978.

41. Lee, E.H. and McMeeking, R.M. "Concerning elastic and plastic components of deformation"; Int Jour Solid Structures; vol 16, p 715-721; Pergamon Press Ltd.; 1980.

42. Lubarda, V.A. and Lee, E.H. "A correct definition of elastic and plastic deformation and its computational significance"; Jour of Applied Mechanics; vol 48, p 35-40; Mar. 1981.

43. Lee, E.H. "Some comments on elastic-plastic analysis"; Int J Solids Structures; vol 17, p 859-872; 1981.

44. Lee, E.H. "Finite deformation effects in plasticity analysis"; Numerical Methods in Industrial Forming Processes; ed. J.F.T. Pittman, R.D. Wood, J.M. Alexander and O.C. Zienkiewicz; Pineridge Press, Swansea, U.K.; July 1982.

45. Fung, Y.C.; Foundations of Solid Mechanics; Prentice Hall Inc.; 1965.

46. Malvera, L.E.; Introduction to the Mechanics of a Continuous Medium; Prentice Hall Inc.; 1969.

47. Cheng, S.Y., Too, J.J.M. and Bera, P.C. "A thermo-elastic-plastic, finite element model for finite deformation of thin shells; to be published.

48. Kirkaldy, J.S., Thomson, B.A. and Bagamis, E.A. "Prediction of multicomponent equilibrium and transformation diagrams for low-alloy steel"; Hardenability Concepts with Applications to Steel; ed. D.V. Doane and J.S. Kirkaldy; ASME; Oct. 1977.

49. Duvall, G.E. and Fowles, G.R. "Shock waves"; J Appl Phys; vol 33; 1962.

50. Carter, W.J. "Relation between dynamic and static phase transformation"; Metallurgical Effects at High Strain Rates; ed. R.W. Rohde, B.M. Butcher, J.R. Holland and C.H. Karnes; Plenum Press; 1973.

51. Lazarus, D. "Intermetallic Diffusion"; Energetics in Metallurgical Phenomena; vol 1; ed. W.M. Mueller; Proc of the 1962 Seminar on Energetics in Metallurgical Phenomena; Gordon and Breach Science Publishers; 1965.

52. Roberts, E.C. "Transformations"; Energetics in Metallurgical Phenomena; vol 1; ed. W.M. Mueller; Proc of the 1962 Seminar on Energetics in Metallurgical Phenomena; Gordon and Breach Science Publishers; 1965.

53. Porter, D.A. and Easterling, K.E.; Phase Transformations in Metals and Alloys; VanNostrand Reinhold Co.; 1981.

54. Stanley, H.E. "Introduction to phase transformations and critical phenomena"; Oxford University Press; 1971.

55. Hillert, M. Prediction of iron-base phase diagrams"; Hardenability Concepts with Applications to Steel; ed. D.V. Doane and J.S. Kirkaldy; 1977.

56. Uhrenius, B. "A compendium of ternary iron-base phase diagram"; Hardenability Concepts with Applications to Steel; ed. D.V. Doane and J.S. Kirkaldy; 1977.

57. Too, J.J.M. and Trafford, R "A finite element method for the analysis of coupled interface problems"; Numerical Methods for Coupled Problems; ed. E. Hinton, P. Bettess and R.W. Lewis; Pinderidge Press; U.K.; 1981.

58. Inoue, T. and Wang, Z.G. "Finite element analysis of coupled thermoinelastic problem with phase transformation"; Numerical Methods in Industrial Forming Processes; ed. J.F.T. Pittman, R.D. Wood, J.M. Alexander and O.C. Zienkiewicz; Pinderidge Press; U.K.; 1982.

59. Kocks, U.F., Argon, A.S. and Ashby, M.F.; Prog Mat Science; vol 19, Pergamon Press; 1975.

60. Sherby, O.D. and Weertman, J.; Acta Met; vol 27; 1979.

61. Burton, B. "Diffusional creep in polycrystalline materials"; Trans Tech Publication; 1977.

62. Pavameswaran, V.R. "Dislocation - Misfit defect interaction in anisotropic diamond cubic crystals"; Materials Science and Engineering; vol 50; 1981.

63. "Electrons in disordered metals and at metallic surfaces"; ed. P. Phariseau, B.L. Gyorffy and L. Scheire; NATO Advanced Study Institutes Series.

64. Hill, R. and Rice, J.R. "Constitutive analysis of elastic-plastic crystals at arbitrary strain"; J Medi Phys Solid; vol 20, p 401-413; Pergamon Press; 1972.

65. Rice, J.R. "On the structure of stress-strain relations for time-dependent plastic deformation in metals"; Transactions of the ASME; J of Applied Mechanics; vol 37; Sept. 1970.

66. Kestin, J. and Rice, J.R. "Paradoxes in the application of thermodynamics to strained solids"; A Critical Review of Thermodynamics; ed. E.B. Stuart, B. Gal-or and A.J. Brainard; Mono Book Corp.; Baltimore; 1970.

67. Rice, J.R. "Continuum plasticity in relation to microscale deformation mechanics"; Metallurgical Effects at High Strain Rate; ed. R.W. Rohde, B.M. Butcher, J.R. Rolland and C.H. Karnas, Sandia Lab.; Plenum Press; New York, NY; 1973.

68. Rice, J.R. "Continuum mechanics and thermodynamics of plasticity in relation to microscale deformation mechanics"; Constitutive Equations in Plasticity; ed. A.S. Argon, MIT Press; 1975.

69. Needleman, A. and Rice, J.R. "Limit ductility set by plastic flow localization"; Mechanics of Sheet Metal Forming; ed. D.P. Koistinen, N.M. Wang; Plenum Press; 1978.

70. Neale, K.W. "Phenomenological constitutive laws in finite plasticity"; SM Archives; vol 6; 1981.

71. Percherski, R.B. "Constitutive modelling for advanced deformations and localization of viscoplastic materials"; Numerical Methods in Industrial Forming Processes; ed. J.F.T. Pittman, R.D. Wood, J.M. Alexander and O.C. Zienkiewicz; Pineridge Press; Swansea, U.K.; July 1982.

72. Asaro, R.J. "Geometrical effects in the Inhomogenous deformation of ductile single crystals"; Acta Metall; vol 27; 1978.

73. Chang, Y.W. and Asaro, R.J. "Lattice rotations and localized shearing in single crystal"; Arch. Mech; vol 32; 1980.

74. Valanis, K.C. "Endochronic theory of plastic fluid (metals under large deformation)"; Application of Numerical Methods to Forming Processes; ed. H. Armen, R.F. Jones, Jr.; ASME; Dec. 10-15, 1978.

75. Valanis, K.C. and Lee, C.F.; private communication.

76. Tirosh, J. and McClintock, F.A. "A numerical approach for solving 2D and anti-plane elasto-plastic problems by assembling dislocations"; Applications of Numerical Methods to Forming Processes; ed. H. Armen and R.F. Jones, Jr.; ASME AMD; vol 28; 1978.

77. Abe, T. and Nagaki, S. "Finite element modelling of Taylor slips in FCC polycrystal deformation of polycrystals"; Deformation of Polycrystals: Mechanisms and Microstructures; ed. N. Hansen, A. Horsewell, T. Leffers and H. Lilholt; 2nd Ris Int Symp on Metallurgy and Material Science; 1981.